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Reducing the Effects of Model Reduction due to Parameter Variations

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Introduction

IN the current literature, most of the methods for model reduction are based upon the assumption that the original system has constant parameters.[1,2] Unfortunately, these methods cannot be applied to systems with parameter variations because the closed-loop system response characteristics may not be acceptable even if the reduced models are stable and accurate for a specific set of constant parameters of the system. Since parameter variations are unavoidable in commonly used physical systems, the objective of this Note is to propose a method for reducing the effects due to parameter variations.

This note extends the authors' method[3] to obtain reduced models that can approximate the original transfer function at $s=0$ and ∞, and at some desirable points on the frequency response curve of the original system. It will be shown subsequently in this Note that by a proper selection of these desirable points, such as the phase-crossover frequency and/or peak point of the frequency response curve, the effects of model reduction on control systems can be reduced with parameter variations.

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The Proposed Method

Let the original transfer function and reduced model be

$$G(s)=\frac{A_{21}+A_{22}s+\ldots+A_{2n}s^{n-1}}{A_{11}+A_{12}s+\ldots+A_{1,n+1}s^{n}} \qquad (1)$$

and

$$R(s)=(\omega_1,\omega_2,\ldots,\omega_m)R[r-1,r]_j^i(s)$$
$$=\frac{d_0+d_1s+\ldots+d_{r-1}s^{r-1}}{c_0+c_1s+\ldots+c_rs^r} \qquad (2)$$

respectively. In Eq. (2), r and $r-1$ represent the numbers of poles and zeros of $R(s)$, respectively; i and j the numbers of terms of the continued-fraction expansion of $G(s)$ about $s=0$ and ∞, respectively; and $\omega_1,\omega_2,\ldots,\omega_m$ the frequencies at which the frequency response of $G(s)$ is matched by that of $R(s)$. The procedure for finding the reduced models is as follows.

Step 1. Expand $G(s)$ about $s=0$ for i (even number) times, i.e.,

$$G(s)=\left[h_1+\left[\frac{h_2}{s}+\left[\ldots+\left[\frac{h_i}{s}+\frac{H_N(s)}{H_D(s)}\right]^{-1}\right]^{-1}\right]^{-1}\right]^{-1} \qquad (3)$$

where $H_N(s)$ and $H_D(s)$ are the numerator and denominator of the remainder of the continued fraction, respectively, and defined as

$$H_N(s)=A_{i+2,1}+A_{i+2,2}s+\ldots+A_{i+2,n-i/2}s^{n-1-i/2} \qquad (4)$$

and

$$H_D(s)=A_{i+1,1}+A_{i+1,2}s+\ldots+A_{i+1,n+1-i/2}s^{n-i/2} \qquad (5)$$

Step 2. Reverse the polynomial sequences in Eqs. (4) and (5), and continue to expand Eq. (3) about $s=\infty$ for j (even number) times; then one has

$$\frac{H_N(s)}{H_D(s)}=\left[E_1s+\left[E_2+\left[\ldots+\left[E_j+\frac{F_N(s)}{F_D(s)}\right]^{-1}\right]^{-1}\right]^{-1}\right]^{-1} \qquad (6)$$

where

$$F_N(s)=A_{i+j+2,n-(i+j)/2}s^{n-1-(i+j)/2}+\ldots$$
$$+A_{i+j+2,2}s+A_{i+j+2,1} \qquad (7)$$

and

$$F_D(s)=A_{i+j+1,n+1-(i+j)/2}s^{n-(i+j)/2}+\ldots$$
$$+A_{i+j+1,2}s+A_{i+j+1,1} \qquad (8)$$

Step 3. Let

$$\frac{T_N(s)}{T_D(s)}=\frac{Y_{m-1}s^{m-1}+y_{m-2}s^{m-2}+\ldots+y_1s+y_0}{x_ms^m+x_{m-1}s^{m-1}+\ldots+x_1s+x_0} \qquad (9)$$

with

$$y_{m-1}=1 \qquad (10)$$

where m is the number of points on the frequency response curve of $G(s)$ to be matched by $R(s)$. Let

$$\left.\frac{T_N(s)}{T_D(s)}\right|_{s=j\omega_k}=\left.\frac{F_N(s)}{F_D(s)}\right|_{s=j\omega_k}=r_k+jm_k \quad k=1,2,\ldots,m \qquad (11)$$

where r_k and m_k are the real and imaginary parts of $F_N(s)/F_D(s)$ for $s=j\omega_k$, respectively. Equating the real and imaginary parts in Eq. (11), one can obtain a set of simultaneously independent equations. Therefore, the $2m$ unknowns $y_{m-2}, y_{m-3}, \ldots, y_1, y_0, x_m, \ldots, x_1, x_0$ can be obtained by solving the $2m$ simultaneously independent equations.

Step 4. Replace $F_N(s)/F_D(s)$ in Eq. (6) by $T_N(s)/T_D(s)$, as defined by Eq. (9), and invert the continued fraction, then the reduced model given in Eq. (2) is obtained, i.e.,

$$(\omega_1,\omega_2,\ldots,\omega_m)R[r-1,r]_j^i(s) = \left[h_1+\left[\frac{h_2}{s}+\left[\ldots+\left[\frac{h_i}{s}+\left[E_1 s+\left[E_2 + \left[\ldots+\left[E_j+\frac{T_N(s)}{T_D(s)}\right]^{-1}\ldots\right]^{-1}\right.\right.\right.\right.\right.\right.\tag{12}$$

8's

The order of the denominator of the reduced model is

$$r=m+(i+j)/2 \tag{13}$$

By use of Eqs . (3) and (6) one has

$$G(s)=\left[h_1+\left[\frac{h_2}{s}+\left[\ldots+\left[\frac{h_i}{s}+\left[E_1 s+\left[E_2+\left[\ldots+\left[E_j+\frac{F_N(s)}{F_D(s)}\right]^{-1}\ldots\right]^{-1}\right.\right.\right.\right.\right.\right.\tag{14}$$

8's

From Eqs. (11), (12), and (14) it can be seen that

$$G(s)=(\omega_1,\omega_2,\ldots,\omega_m)R[r-1,r]_j^i(s) \quad \text{for} \quad s=j\omega_k \qquad k=1,2,\ldots,m \tag{15}$$

Equation (15) indicates that the frequency response of $G(s)$ and $R(s)$ is matched at $s=j\omega_k$, $k=1,2,\ldots,m$. Note that if i and j are odd, Eqs. (3-6) may have some minor differences, but the procedure is the same.

Table 1 Reduced models of $T(s)$

	$R_{11}(s)$ $(\omega_1,\omega_2)R(4,5)_0^6(s)$	$R_{12}(s)$ $(\omega_1,\omega_2)R(3,5)_2^4(s)$
b	-3.0325×10^{-2}	0.686
z_k	$-36.3821\pm j3.7091$ $59.6739\pm j11.8486$	46.2194 -33.0033 -131.1900
p_i	-33.3039 $-0.5004\pm j24.5916$ $-13.1908\pm j49.7827$	-39.7852 $-0.5033\pm j24.5909$ $-9.0891\pm j44.1344$

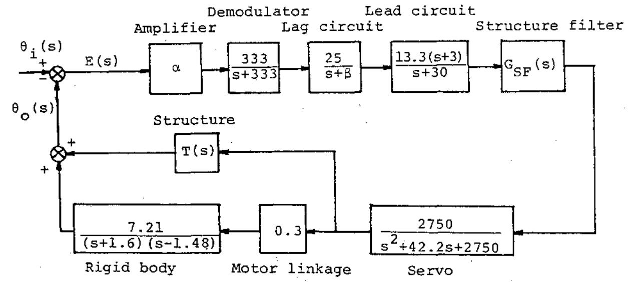

Fig. 1 Block diagram of a flexible rocket control system.

The advantage of the proposed method is that by the suitable selection of matching points on the frequency response curve of the original transfer function, the frequency response at low, intermediate, and high frequencies can be matched; thus, the effects of parameter variations can be reduced.

In comparison with the original continued-fraction method[4,5] the advantage of the proposed method is due to the fact that the remainder $[F_N(s)/F_D(s)$ defined in Eq. (6)] of the continued fraction, which is disregarded in the continued-fraction method, is used to match some desirable points of the frequency response curve.

Application to a System with Parameter Variations

Consider the system shown in Fig. 1.[3] The transfer functions of the structure filter and structure are defined, respectively, as

$$G_{SF}(s)=\frac{(s^2+70s+4000)(s^2+22s+12800)}{(s^2+30s+5810)(s^2+30s+12800)} \tag{16}$$

and

$$T(s)=[0.686(s+53)(s-53)(s^2-152.2s+14500)(s^2+153.8s+14500)]/[(s^2+s+605)(s^2+45.5s+2660)(s^2+2.51s+3900)(s^2+3.99s+22980)] \tag{17}$$

The parameters α and β are defined as

$$15\le\alpha\le20 \tag{18}$$

and

$$40\le\beta\le100 \tag{19}$$

respectively. By use of the proposed method, a number of reduced models of $T(s)$ are obtained with the frequency responses of the reduced models to be matched with that of $T(s)$ at $\omega_1=21$ rad/s and/or $\omega_2=25$ rad/s, where ω_1 is the phase-crossover frequency of the open-loop transfer function $\theta_0(s)/E(s)$ for $\alpha=15$ and $\beta=100$, and ω_2 is the frequency of the peak point on the Bode plot of $T(s)$. The reduced models are shown in Table 1, where p_i, z_k, and b are defined as follows:

$$R(s)=\frac{b\pi(s-z_k)}{\pi(s-p_i)} \tag{20}$$

Since both $T(s)$ and the original system are low-pass in nature, the step-input responses are better for the system with those models having i larger than j. For example, the integral of squared errors of the unit-step responses between the system with $T(s)$ and its reduced models $R_{11}(s)$ and $R_{12}(s)$ are 1.1536×10^{-6} and 1.4612×10^{-6} ($\alpha=15,\beta=100$) and 3.8990×10^{-5} and 6.5488×10^{-5} ($\alpha=20,\beta=40$), respectively.

The method presented in this Note is useful because it can produce useful reduced models by the selection of the numbers of terms of the continued-fraction expansion about $s=0$ and ∞ and the desirable matching points on the frequency response curve of the original system.

Conclusions

A practical method for model reduction has been proposed that can produce reduced models to match the frequency responses of the original system not only at $s=0$ and ∞ but match some desirable points on the frequency response curve of the original transfer function as well. Application of the proposed method to a system with parameter variations has been illustrated.

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A New Algorithm for the Computation of the Geodetic Coordinates as a Function of Earth-Centered Earth-Fixed Coordinates

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Introduction

THERE are many applications that require computation of the geodetic coordinates as a function of Earth-Centered Earth-Fixed (ECEF) coordinates in real time (such as the NAVSTAR Global Positioning System). Therefore, it is highly desirable to find efficient algorithms that can be used in real-time applications. The intent of this Note is to present a new algorithm that is comparable in efficiency to the best described in the literature from the standpoint of the number of elementary operations required per iteration. The proposed method does not require a square-root evaluation within the iterative portion of the algorithm.

Problem Formulation

It is well known that for an ellipsoid of revolution

$$x_e = (r+h)\cos\phi\cos\lambda \tag{1}$$

$$y_e = (r+h)\cos\phi\sin\lambda \tag{2}$$

$$z_e = [(1-e^2)r+h]\sin\phi \tag{3}$$

where

$$r = a/[(1-e^2\sin^2\phi)^{1/2}] \tag{4}$$

where, for a given point (see Fig. 1),

x_e, y_e, z_e = ECEF coordinates
ϕ = geodetic latitude

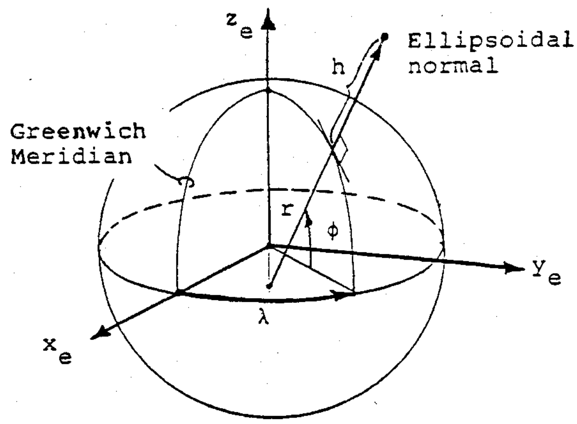

Fig. 1 Geodetic parameters and ECEF frame.

λ = geodetic longitude (measured east from the Greenwich meridian)
h = altitude above reference ellipsoid measured along the normal passing through the point
r = east-west radius of curvature at surface
a = ellipsoidal equatorial radius (a = 6,378,135 m based on Model WGS-72)
e^2 = square of the eccentricity of the reference ellipsoid (e^2 = 0.006694317778 for Model WGS-72)

We seek to determine the geodetic coordinates (ϕ,λ,h) as a function of ECEF coordinates (x_e,y_e,z_e).

To the author's best knowledge, there is at least one direct method[1] and several iterative methods (see Appendix) for the computation of geodetic coordinates as a function of ECEF coordinates. The direct method (unpublished in open literature)[1] requires three square-root operations, two cube-root operations, one inverse tangent evaluation, one sine function evaluation, and about 15 multiplication/division operations. Because the geodetic coordinates are difficult and inefficient to evaluate in closed form, they are usually computed by an iterative method based on a Newton-Raphson technique. A new iterative method is presented in the next section.

Derivation of the Algorithm

First, an iterative process for the computation of the radius of curvature, involving the quantity $v = r^2/a^2$, is derived. Then the altitude and latitude are computed as a function of v.

From Eqs. (1) and (2), it follows that

$$\cos^2\phi = \frac{x_e^2+y_e^2}{(r+h)^2} \tag{5}$$

and Eq. (4) can be written as

$$\sin^2\phi = \frac{r^2-a^2}{e^2r^2} \tag{6}$$

Thus, from Eqs. (5) and (6), we have

$$\frac{r^2-a^2}{e^2r^2} + \frac{x_e^2+y_e^2}{(r+h)^2} = 1 \tag{7}$$

Substituting $\sin\phi$ from Eq. (3) into Eq. (6), the result is

$$\frac{z_e^2}{[(1-e^2)r+h]^2} = \frac{r^2-a^2}{e^2r^2} \tag{8}$$

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